Relaxed CBF-based control for inspection with underactuated USVs

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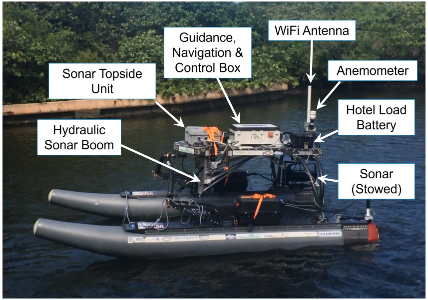
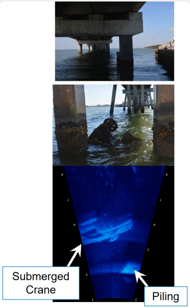
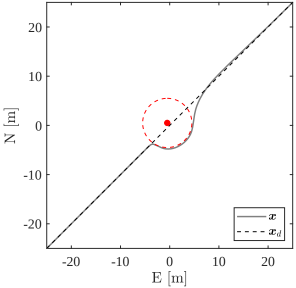
Introduction







Motivation: Survey, monitoring, or interception applications require:

- approaching target as closely as possible,
- while guaranteeing a minimum safe standoff distance,
- global tracking stability with minimal computational effort,
- and dynamically feasible control inputs for underactuated surface vessels.

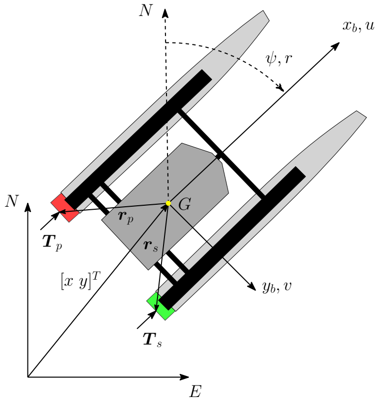
Approach: Use of Control Barrier Functions

Problem Formulation

$$\dot{\boldsymbol{\eta}} = \boldsymbol{J}(\boldsymbol{\eta})\boldsymbol{v},$$
$$\boldsymbol{M}\dot{\boldsymbol{v}} = -\boldsymbol{C}(\boldsymbol{v})\boldsymbol{v} - \boldsymbol{D}(\boldsymbol{v})\boldsymbol{v} + \boldsymbol{\tau}.$$

$$\dot{\boldsymbol{x}} = \begin{bmatrix} u\cos\psi - v\sin\psi \\ u\sin\psi + v\cos\psi \\ r \\ f_x \\ f_y \\ f_\psi \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ \frac{1}{m_{11}} & 0 \\ 0 & \frac{a_y}{m_{33}} \\ 0 & \frac{a_\psi}{m_{33}} \end{bmatrix} \boldsymbol{\tau}$$

$$\boldsymbol{\tau} = [\tau_x \quad \tau_\psi]^T.$$



Control Objective: Stable trajectory tracking and target approach for underactuated USVs.

Candidate Lyapunov Functions

Consider the nonlinear system

$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{G}(\boldsymbol{x})\boldsymbol{u}, \qquad (1)$$

which is affine in the control input.

Lyapunov Stability Theorem

Suppose there is a function

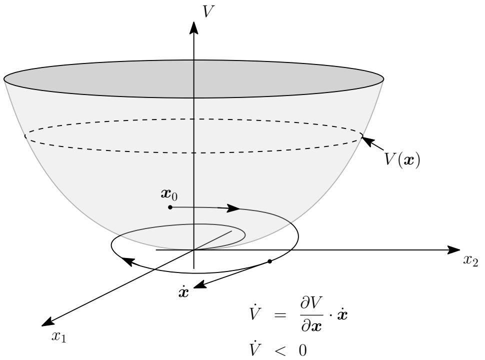
$$V(\boldsymbol{x}) > 0, \quad \forall \boldsymbol{x} \in D, \boldsymbol{x} \neq 0,$$

where

$$\dot{V}(\boldsymbol{x}) < 0, \quad \forall \boldsymbol{x} \in D, \boldsymbol{x} \neq 0,$$

then (1) is asymptotically stable.

Set of points in V shrinks in time since $\dot{V} < 0$.



Standard Control Barrier Functions

Safe Set (convex)

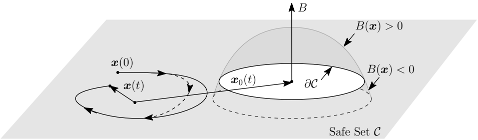
$$\begin{aligned}
\mathcal{C} &= \{\boldsymbol{x} \in D \subset \mathbb{R}^n : B(\boldsymbol{x}) \geq 0\}, \\
\partial\mathcal{C} &= \{\boldsymbol{x} \in D \subset \mathbb{R}^n : B(\boldsymbol{x}) = 0\}, \\
\mathrm{Int}(\mathcal{C}) &= \{\boldsymbol{x} \in D \subset \mathbb{R}^n : B(\boldsymbol{x}) > 0\},
\end{aligned}$$

Control Barrier Function

B is a CBF if

$$\sup_{u \in \mathcal{U}} \left[\frac{\partial B}{\partial \boldsymbol{x}} \boldsymbol{f} + \frac{\partial B}{\partial \boldsymbol{x}} \boldsymbol{G} \boldsymbol{u} \right] \geq -\alpha(B(\boldsymbol{x})) \quad (2)$$

Use of $\alpha(\cdot)$ (a $\mathcal{K}_\infty$ function) keeps safe set from shrinking.



Theorem: If $B(\boldsymbol{x})$ is a CBF, any locally Lipschitz continuous u satisfying (3) renders $\mathcal{C}$ safe for (1).

Relaxed Control Barrier Functions

Consider system (1) and safe set $\mathcal{C}$. A class C^1 function $B : D \times D \to \mathbb{R}$, where $D \subset \mathbb{R}^n$, is a relaxed control barrier function if:

1. $B(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{x}}_0) \geq 0$ for all $\tilde{\boldsymbol{x}}_0 \in \mathcal{C}$.
2. B is proper, i.e. $\{\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{x}}_0 \mid B(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{x}}_0) \leq L\}$ is compact for any $L \geq 0$.
3. For any continuous $\boldsymbol{u}_c \in \mathbb{R}^m$, there exist non-negative constants $\alpha, \beta \geq 0$, so that

$$\begin{aligned} \inf_{\boldsymbol{u}_c \in \mathbb{R}^m} \dot{B} &= \inf_{\boldsymbol{u}_c \in \mathbb{R}^m} \left[\frac{\partial B}{\partial \tilde{\boldsymbol{x}}} \cdot \dot{\tilde{\boldsymbol{x}}} + \frac{\partial B}{\partial \tilde{\boldsymbol{x}}_0} \cdot \dot{\tilde{\boldsymbol{x}}}_0 \right], \\ &< \alpha B(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{x}}_0) + \beta. \end{aligned} \tag{3}$$

The trajectory tracking safety-critical control input is designed by solving the optimization problem

$$\min_{\boldsymbol{u}_c} \|\boldsymbol{u}_c - \boldsymbol{u}_t\|^2, \tag{4a}$$

$$\text{s.t.} \quad \dot{B} - \alpha B(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{x}}_0) - \beta < 0. \tag{4b}$$

Trajectory Tracking Safety-Critical Controller

Consider system (1). Let $\boldsymbol{u}_t$ be a trajectory tracking control input,

$$I := \left(\frac{\partial B}{\partial \tilde{\boldsymbol{x}}} + \frac{\partial B}{\partial \tilde{\boldsymbol{x}}_0}\right) \cdot [\boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{g}(\boldsymbol{x})\boldsymbol{u}_t] - \frac{\partial B}{\partial \tilde{\boldsymbol{x}}} \cdot \dot{\boldsymbol{x}}_d - \frac{\partial B}{\partial \tilde{\boldsymbol{x}}_0} \cdot \dot{\boldsymbol{x}}_0, \tag{5}$$

$$L_g B := \left(\frac{\partial B}{\partial \tilde{\boldsymbol{x}}} + \frac{\partial B}{\partial \tilde{\boldsymbol{x}}_0}\right) \cdot \boldsymbol{g}(\boldsymbol{x}), \tag{6}$$

and

$$J := \alpha B(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{x}}_0) + \beta. \tag{7}$$

The control input

$$\boldsymbol{u}_c = \begin{cases} \boldsymbol{u}_t, & I \leq J, \\ \boldsymbol{u}_t - (I - J)\dfrac{L_g B^T}{\|L_g B\|^2}, & I > J, \end{cases} \tag{8}$$

is the solution of the optimal control Problem (4).

Control Design: Underactuated USV

Tracking errors: $\tilde{x} = x - x_d$ and $\tilde{y} = y - y_d$

Distance to target: $\tilde{x}_0 := x - x_0$ and $\tilde{y}_0 := y - y_0$

Approach:

1. underactuated USV can apply both a surge force and yaw moment, but cannot directly apply a force in the sway direction
2. motion in the sway and yaw directions coupled via the added mass terms
3. use coupling to find yaw moment τ_ψ required for virtual control input $\dot{r}_c$ that relates yaw acceleration to the sway acceleration, such that commanded control inputs dynamically feasible.
4. control design includes two backstepping stages + Dynamic Surface Control
   - design kinematic position tracking controller $\rightarrow$ virtual control inputs
   - design physical control inputs taking nonholonomic second order (acceleration) constraints into account
   - use DSC to compute time derivatives of virtual control inputs

Control Design: Kinematic Controller

Take virtual control inputs to be

$$\dot{x}_t = -k_x \tilde{x} + \dot{x}_d \tag{9}$$

and

$$\dot{y}_t = -k_y \tilde{y} + \dot{y}_d, \tag{10}$$

where $k_x > 0$ and $k_y > 0$ are constants.
Trajectory tracking control input (virtual) is given by $\boldsymbol{u}_t = [u_t \quad v_t]^T$, where

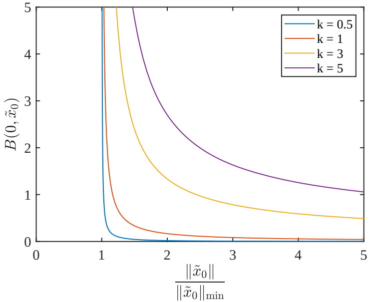
$$\begin{aligned} u_t &= \dot{x}_t \cos\psi + \dot{y}_t \sin\psi, \\ v_t &= -\dot{x}_t \sin\psi + \dot{y}_t \cos\psi. \end{aligned} \tag{11}$$

Control Design: Safety-Critical Kinematic Controller

Use modified form of relaxed CBF proposed by Igarashi & Nakamura (2018), take

$$B(\tilde{\mathbf{x}}, \tilde{\mathbf{x}}_0) = \frac{1}{2}\left\{ \frac{1}{\left[\|\tilde{\mathbf{x}}_0\|^{1/k} - \|\tilde{\mathbf{x}}_0\|^{1/k}_{\min}\right]} + \tilde{\mathbf{x}}^T\tilde{\mathbf{x}} \right\}, \tag{12}$$

where k selected to modify agressiveness of safety critical controller as the USV approaches safety "barrier" located at $\|\tilde{\mathbf{x}}_0\| = \|\tilde{\mathbf{x}}_0\|_{\min}$.



Control Design: Safety-Critical Kinematic Controller

Kinematic equation of motion for the virtual control inputs u_c and v_c is

$$\begin{bmatrix} \dot{x}_c \\ \dot{y}_c \end{bmatrix} = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix} \begin{bmatrix} u_c \\ v_c \end{bmatrix}. \tag{13}$$

Virtual tracking control inputs are in the control affine form of (1) with $\boldsymbol{f}(\boldsymbol{x}) = 0$ and $\boldsymbol{G} = \boldsymbol{g}(\psi)$, where

$$\boldsymbol{g}(\psi) = \begin{bmatrix} \cos\psi & -\sin\psi \\ \sin\psi & \cos\psi \end{bmatrix}. \tag{14}$$

Control Design: Dynamic safety-critical tracking control

To determine physical control inputs τ_x and τ_ψ that generate virtual control input $\boldsymbol{u}_c$, define velocity error surfaces $\tilde{u} := u - u_c$, $\tilde{v} := v - v_c$ and $\tilde{r} := r - r_c$.
Take virtual control input $\dot{r}_c$ and physical control inputs τ_x and τ_ψ to be

$$\begin{aligned} \dot{r}_c &= \frac{a_\psi}{a_y}\left[-k_v\tilde{v} - f_y + \dot{v}_c\right] + f_\psi, \\ \tau_x &= m_{11}\left[-k_u\tilde{u} - f_x + \dot{u}_c\right], \\ \tau_\psi &= \frac{m_{33}}{a_\psi}\left[-k_r\tilde{r} - f_\psi + \dot{r}_c\right], \end{aligned} \tag{15}$$

where $k_u > 0$, $k_v > 0$ and $k_\psi > 0$ are constants.

Control Design: Dynamic Surface control

$\hat{u}_c$ and $\hat{v}_c$ filtered estimates of u_c and v_c

Let $\tilde{u} := u - \hat{u}_c$, $\tilde{v} := v - \hat{v}_c$, $\tilde{u}_c := \hat{u}_c - u_c$ and $\tilde{v}_c := \hat{v}_c - v_c$.

Use exact expression for $\dot{r}_c \rightarrow$, $\tilde{r} := r - r_c$, as before.

Now, take the virtual control input $\dot{r}_c$ and physical control inputs τ_x and τ_ψ to be

$$\begin{aligned} \dot{r}_c &= \frac{a_\psi}{a_y}\left[-k_v\left(v - v_c\right) - f_y + \dot{\hat{v}}_c\right] + f_\psi, \\ \tau_x &= m_{11}\left[-k_u\left(u - u_c\right) - f_x + \dot{\hat{u}}_c\right], \\ \tau_\psi &= \frac{m_{33}}{a_\psi}\left[-k_r\left(r - r_c\right) - f_\psi + \dot{r}_c\right]. \end{aligned} \tag{16}$$

Closed loop error system for the dynamics of the system is

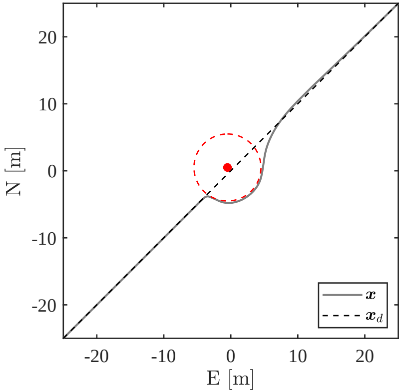
$$\begin{aligned} \dot{\tilde{u}} &= -k_u\tilde{u} - k_u\tilde{u}_c, \\ T_d\dot{\hat{u}}_c &= -\tilde{u}_c, \\ \dot{\tilde{v}} &= -k_v\tilde{v} - k_v\tilde{v}_c, \\ T_d\dot{\hat{v}}_c &= -\tilde{v}_c, \\ \dot{\tilde{r}} &= -k_r\tilde{r}, \end{aligned} \tag{17}$$

where $T_d \in (0, 1)$ is filter time coefficient of the filter and $\hat{u}_c(0) = 0$ and $\hat{v}_c(0) = 0$.

Representative Simulations

- Desired trajectory straight-line constant speed 1.4 m/s
- Control parameters manually tuned

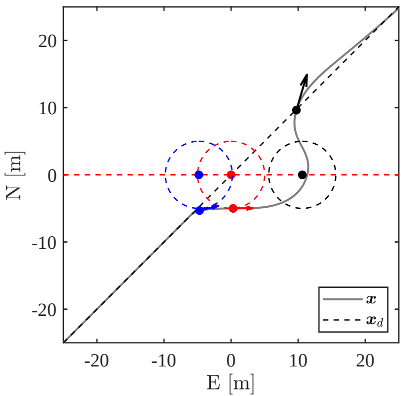
Parameter	**Value**
α	0.75
β	1
k	5
k_x, k_y	0.5
k_u, k_v, k_r	0.075
a	0.5
$\lVert \boldsymbol{x}_0 \rVert_{\min}$	5.0
T_d	0.01

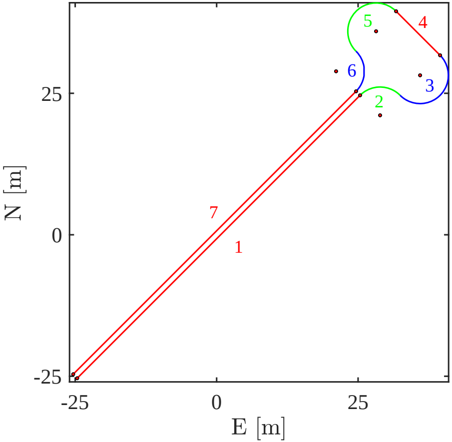


Representative Simulations

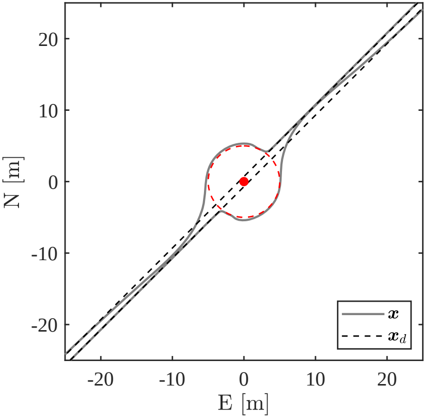
- Desired trajectory straight-line constant speed 1.4 m/s
- (Same) manually tuned control parameters

Parameter	Value
α	0.75
β	1
k	5
k_x, k_y	0.5
k_u, k_v, k_r	0.075
a	0.5
$\|\boldsymbol{x}_0\|_{\min}$	5.0
T_d	0.01



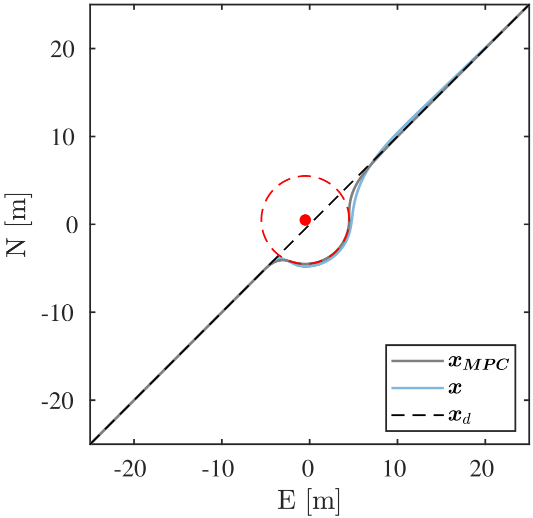


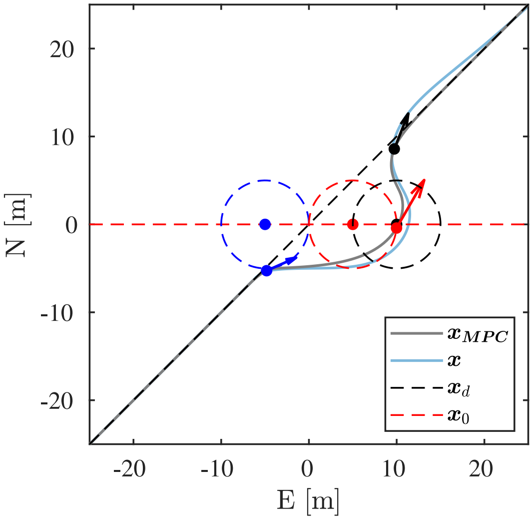
N [m]
E [m]
25
0
-25
-25
0
25
1
2
3
4
5
6
7



N [m]
E [m]
20
10
0
-10
-20
-20
-10
0
10
20
$\boldsymbol{x}$
$\boldsymbol{x}_d$

Comparison with Nonlinear MPC





Concluding Remarks

- Proposed trajectory tracking safety-critical controller for closest safe approach by an underactuated USV with nonholonomic dynamic (acceleration) motion constraints
- analytical solution to the optimization problem, instead of online optimization – computationally lightweight
- modification of relaxed control barrier function of Igarashi & Nakamura (2018), permits the safety critical control to start acting sooner and more gradually
- not as smooth/precise as MPC, but much simpler to configure and less computationally intense
- Future work:
  - Backstepping leads to a nonlinear PD controller – opens possibility of robust approaches
  - reduce actuator saturation when the safety-critical controller is deactivated
  - extend the proposed approach to handle model uncertainty, exogenous disturbances and multiple targets